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A MILP approach for detailed operational scheduling of a supply chain in the phosphate industry

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Résumé – L'ordonnancement à court terme des opérations d'une chaîne d'approvisionnement dans l'industrie de phosphate est un problème complexe avec de nombreuses contraintes spécifiques à l'industrie. Nous nous sommes intéressés à la résolution d'un problème d'ordonnancement intégré de production (stations de lavage) et de transport via un pipeline multiproduits. Dans cet article, une approche basée sur un modèle de programmation linéaire mixte en nombres entiers avec une formulation en temps continu est proposée. L'objectif est de déterminer un ordonnancement de la chaîne d'approvisionnement permettant de maximiser le taux d'utilisation du pipeline, tout en respectant les contraintes liées à la production, au transport, à la capacité de stockage, et à la satisfaction des demandes de clients. Un groupe d'instances à court terme ont été générées à partir des données réelles de la chaîne d'approvisionnement de l'OCP et les résultats obtenus montrent que le modèle est capable de fournir des solutions optimales de ces instances dans un délai raisonnable.

Abstract –Short-term detailed multiproduct phosphate production, pipeline transportation, and storage management are complex problems with many industry-specific constraints. We are interested in solving an integrated production (washing stations) and multiproduct pipeline transportation scheduling problem in the context of (phosphate) mining industry. In this paper, an approach based on a mixed-integer linear programming model with a continuous-time formulation is proposed. The objective is to determine a scheduling of the supply chain allowing to maximize the utilization rate of the pipeline, while respecting the constraints related to production, transport, storage capacity, and satisfaction of customers' demand requests. A group of short-term instances were generated using the real dataset of OCP's supply chain and the obtained results show that the model is able to provide efficiently optimal solutions for these instances.

***Mots clés* –** ordonnancement, chaîne d'approvisionnement, pipeline multiproduit, modèle linéaire, industrie du phosphate.
***Keywords* –**scheduling, supply chain, multiproduct pipeline, MILP model, phosphate industry.

1 INTRODUCTION

In this paper, we propose a mixed-integer linear programming (MILP) solution approach for the integrated multiproduct production and pipeline transportation scheduling problem in the context of the phosphate industry. Phosphate ore is crushed and washed at the Washing Stations (production units) to create a mixture of ore and water (slurry) before to be transported to clients through the pipeline. Intermediate limited capacity storage tanks are used. The slurry pipeline is a cost-effective and reliable option for transporting large quantities of minerals over long distances. By utilizing pipelines, it is possible to maintain a continuous supply of slurry without the need for oversized storage units, as is required for traditional transportation modes such as rail and truck. With the use of multiproduct limited storage capacity tanks, the scheduling problem becomes more challenging.

This study aims to determine the duration of every time interval, the production decisions, and the charging and discharging decisions at each time interval, to schedule the transportation of batches through the pipeline. According to the Bone Phosphate Lime (BPL) component, phosphate slurries are classified into different qualities, and each of them are considered as different products. A batch of phosphate slurry is composed of a single product with lower and upper volume constraints. A batch of water is placed between two batches of different products to prevent contamination and enable the identification of batch edges at the end of the pipeline. Additionally, the slurry pipeline is required to be injected with either a product or water at all times, meaning no stoppages are allowed. Washing stations are used to produce phosphate slurry and are characterized by a minimum production quantity to be respected and a setup duration in case of a product change.

Considering a set of products, a dominant product P_1 ensures the continuous functioning of the chemical processing units. P_1 is discharged into a tank with a maximum autonomy of four hours in cases of low demand, requiring careful management of storage to accurately represent the real-world situation. For the other products, a tank is shared between them for intermediate storage at the Head station and the Terminal station. The tank can contain only one product at a time. The change of stored product in the tanks requires additional constraints to avoid contamination. The products are filtered to reduce water content and consumed before the end of the scheduling horizon.

The scheduling horizon is divided into a predefined number of time intervals. A mixed-integer linear programming model is proposed with the goal of maximizing the use of the slurry pipeline to transport products. The initial storage levels, initial stored products in tanks, and batches contained in the pipeline at the start of the scheduling horizon are considered in the model.

The rest of the paper is organized as follows. The next section defines the problem under study and assumptions. Section 3 presents a literature review on the multi-products unidirectional pipeline scheduling problems. In Section 4, the proposed MILP model is presented. Section 5 presents the computational results. Finally, Section 6 presents conclusions and future research which may result from this work.

2 PROBLEM DEFINITION

The problem studied in this paper is illustrated in Figure 1. Phosphate ore is processed in the washing stations to create a product slurry. The phosphate ore slurry is either stored in a tank at the washing station if the flow rate of production is different from the transportation flow rate of the related secondary pipeline (case of washing station W_1), or to be send directly to the head station. In the head station, the tank $V_{2,1}$ is dedicated to the storage of the primary product P_1, which accounts for over 90% of the total demand. The other products can be stored in the tank $V_{2,2}$. The slurry pipeline transports product batches from the head station to the terminal station. In the terminal station $V_{3,1}$ is dedicated to the storage of P_1 and $V_{3,2}$ is used for the storage of the other products that are then processed by the filtration unit. Based on the demand, the objective is to provide a tailed scheduling of batches in the washing stations, secondary pipelines and the slurry pipeline while maximizing the utilization rate of the slurry pipeline. Furthermore, detailed storage management is needed to ensure respecting storage capacity and contamination constraints.

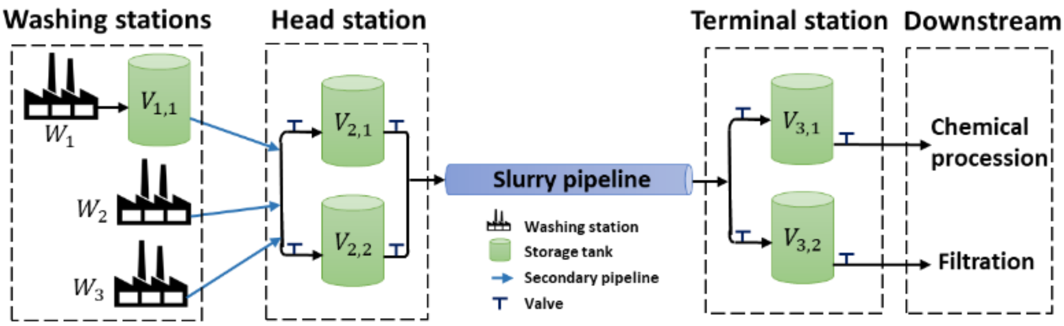

Figure 1. The scheduling problem schematic overview

The following inputs settings are known:

- the number of products.
- the number of time intervals in the scheduling horizon.
- the maximum number of new batches that can be charged in the slurry pipeline during the scheduling horizon.
- the length of the scheduling horizon.
- the flow rate of pipelines, the production rate of washing stations, and the processing flow rate of the filtration station.
- the upper and lower bounds of batch sizes.
- The setup duration of the washing stations and the minimal shutdown duration for the secondary pipelines between the washing stations and the head station.
- the continuous consumption flow rate of P_1 and the overall demand for the other products at the filtration station.
- the set of allowed products to be scheduled in the washing stations, secondary pipelines, and tanks.
- the initial state of the slurry pipeline: (old) batches charged in the slurry pipeline at the beginning of time horizon.
- the initial inventory levels and products present in the tanks.

The following assumptions are considered:

- the pipeline is always full, and the contained products are incompressible.
- the flow rate of the slurry pipeline is uniform.
- only one product can be stored in a tank at a given time interval.
- only one product can be charged in the slurry pipeline at a given time interval.
- only one product can be scheduled in a washing station at a given time interval.
- all tanks contain an initial quantity of a given product at the start of the scheduling horizon.
- the stoppage of the slurry pipeline is not allowed. In case of the saturation of storage capacity at terminal station, a volume of water will be injected in the slurry pipeline.
- intermediate storage tank is not considered at washing stations if the pumping rate of the related secondary pipeline is equal to that of production.
- the washing stations W_1 and W_2 are dedicated for the production of P_1.

3 LITERATURE REVIEW

In previous years, pipeline scheduling problems have gained increased attention due to the potential for operational cost savings. Most of existing works in the literature on this topic focuses on the petroleum industry, with fewer published works addressing scheduling problems in the context of mining industry. These problems can vary in complexity and can be classified based on pipeline structure and its number of sources and destinations (Magatão et al., 2015): straight pipeline, tree-structured pipeline, and pipeline network. Furthermore, pipeline scheduling problems are NP-complete, as demonstrated by

(Jittamai, 2004) for a simple problem of multi-product scheduling of a single-source pipeline subject to delivery time windows.

This paper is specifically interested in the multiproduct slurry pipeline scheduling problem, the washing stations scheduling problem, and the multiproduct storage management problem in the context of the phosphate mining industry. A set of secondary pipelines is also considered. The previous works on pipeline scheduling can be divided into two main categories based on the time formulation used: continuous time and discrete time formulations.

The continuous-time formulation, which divides the scheduling horizon based on the start and end time of each task, is the most used in the literature. Works such as (Relvas et al., 2006; Relvas et al., 2009; Cafaro and Cerdá, 2008; MirHassani and BeheshtiAsl, 2013; Moradi and MirHassani, 2016; Chen et al., 2019; Bamoumen et al., 2023) have employed mixed-integer linear programming (MILP) models with a continuous time formulation to address pipeline scheduling. The continuous time formulation models are generally successful in minimizing the scale and complexity of the problem and can be used in addition to heuristic methods to overcome computational difficulties.

(Relvas et al., 2006) uses a MILP approach for a multiproduct one-to-one pipeline scheduling and inventory management problem where storage levels are balanced daily during a one-month scheduling time horizon. The model is initialized with either a fixed sequence of batches, a part of the sequence is fixed (mixed sequence) or all the sequence is free. The MILP is able to reach feasibility for the fixed and mixed sequences but requires decomposing the scheduling horizon into two periods (15 days each) and solving two MILP problems for free sequences. A solution for generating desirable sequences was proposed by (Relvas et al., 2009), where a heuristic can compute initial fixed sequences for the MILP model. The final scheduling solution was tested on three real scenarios varying on the length of the scheduling horizon.

(Bamoumen et al., 2023) uses a MILP model, focusing on the discharging time axis, to provide a scheduling solution for the multi-product straight unidirectional pipeline scheduling problem. In addition, a GRASP-like algorithm, using a construction method and an improvement procedure, is used to solve the same problem. The GRASP-like algorithm was compared to the MILP model, proving its ability to provide competitive results both in terms of solution quality and CPU time.

(Chen et al., 2019) uses a generalized MILP approach for pipeline network scheduling. The number of time intervals and the number of batches are given as input. The objective of the model is to minimize the makespan while providing the length of every time interval and a single charging and discharging decision for every pipeline during every time interval. In addition, the storage level of the tanks is updated at the end of every time interval. Therefore, the proposed model can ensure better storage management than models where storage levels are calculated during a number of predefined time stamps. The results were compared with instances from the literature such as from (Cafaro and Cerdá, 2012), proving the model to be versatile in solving a range of pipeline network scheduling problems in a reasonable CPU time.

While most works in the literature consider the due date of product consumption at the end of the scheduling horizon, in real-world problems demand can vary during the scheduling horizon. Therefore, dynamic models with flexible demand representations are more useful, but also more complex than static models where demand is linearized over the duration of the scheduling horizon. Furthermore, in the case of a limited storage autonomy, combined with a continuous consumption flow rate, a more rigorous storage management is needed.

(Moradi and MirHassani, 2016) proposes an approach to take demand uncertainty into consideration. A deterministic demand is used by a MILP model, then, a Γ- robustness approach is used to extend the model to a robust formulation. Results prove the feasibility of most of the scenarios studied.

The discrete-time formulation aims to divide the scheduling horizon into multiple time intervals and the pipeline into multiple packages, each containing a single product. Works such as (Shah, 1996; Relvas et al., 2013; Sidki et al., 2022) have used MILP models and decomposition approaches with this formulation to address pipeline scheduling problems. (Relvas et al., 2013) used two MILP models: The Fixed Batch Size using predefined batch sizes based on the scheduled products and the Variable Batch Size model introduces an upper and lower bound for batch sizes. The two models were compared demonstrating that the Variable Batch Size model provides more flexible solutions using less CPU time. (Sidki et al., 2022) proposed a mixed-integer linear programming model with a discrete-time formulation to solve a storage-sensitive slurry pipeline scheduling problem. The proposed model aims to minimize the total products sold-out and the total water volume charged in the slurry pipeline. The model was tested on different instances of a scheduling horizon of 2 days, demonstrating it can achieve optimality in few seconds.

The model proposed in this paper makes use of some pipeline scheduling principles presented by (Chen et al., 2019) and the overall literature, specialty for the slurry pipeline scheduling. A predetermined number of time intervals is given in advance. The objective is to maximize the use of the slurry pipeline transporting products while providing for every time interval, the decision to be made for the charging and discharging operations in the slurry pipeline, the decision to be made in every secondary pipeline and washing station, and providing detailed storage management while considering the capacity constraints of the tanks and avoiding contamination.

The contribution of this work to the literature is summarized as follows:

- A detailed scheduling solution for the washing stations and the secondary pipelines. In the literature, the refineries are generally simplified to a continuous production flow rate for all products.
- A solution for managing multiproduct tanks able to store different products.
- A total discharge of the slurry pipeline at the end of the scheduling horizon to observe the impact of scheduling decisions on demand and tanks in the terminal station.

Nomenclature

Sets and indexes

- $i \in I = \{1, \ldots, IM\}$: set of batches transported by the slurry pipeline. $IM = IOM + INM$, IOM is the number of old batches and INM is the number of new batches.
- $IO = \{1,2, \ldots, IOM\}$: set of old batches in the slurry pipeline at the start of a scheduling horizon, numbered from the farthest to closest to the origin of the pipeline.
- $IN = \{IOM + 1, \ldots, INM\}$: set of new batches to be charged in the slurry pipeline during the scheduling horizon.
- $j \in J_n = \{1,2\}$: set of stations along the slurry pipeline. $j = 1$ represent the head station and $j = 2$ represent the terminal station.

- $k \in K = \{1, \dots, KM\}$: set of time nodes. KM is the number of time nodes for the charging operations of the slurry pipeline.
- $m \in M = \{1, \dots, MM\}$: set of stations. MM indicates the total number of stations containing storage tanks.
- $p \in P = \{1, \dots, PM\}$: set of products. PM indicates the total number of products.
- $cp \in CP = \{1, \dots, CPM\}$: set of products defined by a continuous consumption flow rate.
- $dp \in DP = \{CPM + 1, \dots, PM\}$: set of products defined by a total quantity to be satisfied at the end of the scheduling horizon.
- $s \in S_m = \{1, \dots, SM_m\}$: set of tanks at every station m.
- $q \in WS = \{1, \dots, WM, \dots, WM + FM\}$: set of washing stations and secondary pipelines. WM indicates the total number of washing stations and FM indicates the total number of secondary pipelines.
- $a \in CU_q$: set of allowed decisions to take place in the washing station q. CU_q contains the products that can be processed by q and $a = PM + 1$ reffers to a shutdown decision.
- $b \in CS_{m,s}$: Set of products allowed be stored in the tank s in the station m.

Continuous Parameters
- F: volume of the slurry pipeline (m^3).
- QIA: the flow rate of the slurry pipeline (m^3/h).
- FFU: maximum flow rate of the filtration unit.
- $QR_{q,p}$: The processing flow rate of product p in q.
- TL: length of the scheduling horizon of the charging operations of the slurry pipeline.
- $VBPA_p$, $VBPI_p$: maximum and minimum sizes of a product batch $p \in P$ or a batch of water ($p = PM + 1$) charged into the slurry pipeline.
- $VMTA_{m,s}$, $VMTI_{m,s}$: maximum and minimum allowed inventory levels in tank s at the station m.
- $VMTO_{m,s}$: initial inventory in the tank s in the station m.
- $VMPD_p$: for $p \in CP$, $VMPD_p$ is the consumption flow rate of product p. for $p \in DP$, $VMPD_p$ is the overall demand quantity of product p.
- WO_i: initial volume of old batch i in the slurry pipeline.
- $VBWS_{q,p}$: minimum size of batch of product p allowed in the washing stations or the secondary pipeline q.
- WST_q: washing stations q setup deration or minimal stoppage duration for the secondary pipeline q.
- τ: Minimum length of a time interval.
- M: A large constant.

Binary Parameters
- $FB_{p,p'}$: 1 if product or water p and product or water p' are allowed to be adjacently transported in the slurry pipeline.
- $YO_{i,p}$: 1 if old batch $i \in IO$ in the slurry pipeline consists of product $p \in P$ or water ($p = PM + 1$).
- $AS_{q,m,s}$: 1 if the washing station or secondary pipeline q is linked to the tank s in the station m
- $OS_{m,s,p}$: 1 if tank s in the station m contain product p at the start of the scheduling horizon.

Continuous variables
- t_k: a time node (date of a given event).
- l_k: length of time interval (t_k, t_{k+1}), $l_k = t_{k+1} - t_k$.
- $h_{i,k}$: upper volumetric coordinate of batch i in the slurry pipeline at t_k.
- $lx_{j,i,k}$: The duration of charging into ($j = 1$) or discharging from ($j = 2$) batch i during (t_k, t_{k+1}).
- $v_{j,i,k}$: volume of batch i that station j charges into or discharges from the slurry pipeline during (t_k, t_{k+1}).
- $vp_{j,i,p,k}$: volume of product p from batch i that the slurry pipeline charges from or discharges into station j during (t_k, t_{k+1}).
- $vmcp_{p,k}$: volume of product p that the terminal station supplies to its client during (t_k, t_{k+1}).
- $vmt_{m,s,k}$: inventory level in the tank s in station m at t_k.
- $w_{i,k}$: volume of batch i in the slurry pipeline at t_k.
- $bw_{q,a,k}$: the processing duration of a scheduling decision $a \in CU_q$ in unit q during (t_k, t_{k+1}).
- $lw_{q,a,k}$: the total processing duration of the continuous scheduling of decision $a \in CU_q$ in unit q up to the time interval (t_k, t_{k+1}).
- $lbw_{q,a,k}$: variable used to linearize the product of $yw_{q,a,k}$ and $bw_{q,a,k-1}$.

Binary variables
- sp_k: 1 the slurry pipeline is active during (t_k, t_{k+1}).
- $x_{j,i,k}$: 1 if the station j charges into batch i or discharges from batch i during (t_k, t_{k+1}).
- $y_{i,p}$: 1 if batch i in the slurry pipeline contains product p.
- $qst_{m,s,b,k}$: 1 if tank s contain product b at t_k in the station m. $qst_{m,s,b,k}$ is defined for only the multiproduct tanks, meaning $|CS_{m,s}| > 1$.
- $yw_{q,a,k}$: 1 if unit q processes $a \in CU_q$ during (t_k, t_{k+1}).
- $ew_{q,a,k}$: 1 if the time interval (t_k, t_{k+1}) coincides with the end of the processing of $a \in CU_q$ in q (the end of a product batch or a shutdown).
- $avp_{m,s,k}$: 1 if the tank s in the station m is empty at moment t_k. $avp_{m,s,k}$ is defined if $|CS_{m,s}| > 1$.

These variables and the previously defined parameters are used in the mixed-integer linear programming model (MILP) presented in the next section.

4 MATHEMATICAL MODEL

4.1 Objective function

The objective function is to maximize the use of the slurry pipeline for transporting products during the scheduling horizon (t_1, t_{KM}). $QIA \times TL$ is the total capacity of the slurry pipeline and $\sum_{i \in \{IOM-1 \dots, INM\}} \sum_{k \in \{1,\dots,KM-1\}} vp_{1,i,PM+1,k}$ is the total volume of water charged during the scheduling horizon.

$$Maximize\ Z \tag{1}$$

where $Z = \left(\frac{QIA \times TL - \sum_{i \in \{IOM-1 \dots, INM\}} \sum_{k \in \{1,\dots,KM-1\}} vp_{1,i,PM+1,k}}{QIA \times TL}\right)$.

4.2 Time constraints

The beginning of the scheduling horizon is formulated as the first-time node. The last time node for the charging operations in the slurry pipeline must be equal to the length of the scheduling horizon TL. After t_{KM} the content of the slurry pipeline is discharged/emptied until t_{KM+INM}. $\frac{F}{QIA}$ indicates the total transportation time in the slurry pipeline.

$$t_1 = 0 \tag{2}$$

$$t_{KM} = TL \tag{3}$$

$$t_{KM+INM} = TL + \frac{F}{QIA} \tag{4}$$

Constraint (5) states the relation between two consecutive time nodes. For the time intervals in $(l_{KM}, \dots, l_{KM+IN-1})$ the batches in IN, contained in the slurry pipeline, are discharged.

$$t_{k+1} = t_k + l_k \quad \forall\, k \in \{1, \dots, KM + INM - 1\} \qquad (5)$$

A lower bound τ is considered for every time interval l_k. τ is relative the minimal charging duration of a batch of water in the slurry pipeline.

$$l_k \geq \tau \;\forall\, k \in \{1, \dots, KM + INM - 1\} \qquad (6)$$

4.3 *Washing stations and secondary pipelines*

During every time interval, only one decision $a \in CU_q$ (production or a shutdown) should be taken in a washing station q as shown in equation (7).

$$\sum_{a \in CU_q} yw_{q,a,k} = 1 \;\forall\, q \in WS, k \in \{1, \dots, KM - 1\} \qquad (7)$$

After every product change in a washing station q, a station shutdown needs to be scheduled. In equation (8) if $yw_{q,a,k-1} = 1$, either the same product $a \in CU_q \backslash \{PM + 1\}$ needs to be processed during (t_k, t_{k+1}) $(yw_{q,a,k} = 1)$ or a shutdown needs to take place during (t_k, t_{k+1}) $(yw_{q,PM+1,k} = 1)$.

$\forall\, q = \{1,2, \dots, WM\}, a \in CU_q \backslash \{PM + 1\}, k \in \{1, \dots, KM - 1\}$:

$$yw_{q,a,k-1} \leq yw_{q,a,k} + yw_{q,PM+1,k} \qquad (8)$$

Based on the scheduling decision $yw_{q,a,k}$ of the washing stations or the secondary pipelines q, for $a \in CU_q$ and during (t_k, t_{k+1}). $bw_{q,p,k}$ is calculated as the product of $yw_{q,p,k}$ and l_k. Therefore, if $bw_{q,a,k} = l_k$ if only if $yw_{q,a,k} = 1$. Otherwise, $bw_{q,a,k} = 0$.

$\forall\, q \in WS, a \in CU_q, k \in \{1, \dots, KM - 1\}$:

$$yw_{q,a,k} = 1 \rightarrow bw_{q,a,k} = l_k \qquad (9)$$

$$yw_{q,a,k} = 0 \rightarrow bw_{q,a,k} = 0 \qquad (10)$$

The duration of a continuous production or a continuous shutdown until a given time interval (t_k, t_{k+1}) in a unit q is determined as to be able to respect the minimal batch size constraints and the minimal setup duration constraints. $lw_{q,a,k}$ represents the cumulative duration of taking the scheduling decision $a \in CU_q$ up to the time interval (t_k, t_{k+1}) for a unit q. The variable $lbw_{q,a,k}$ helps to verify if the scheduling decision $a \in CU_q$ is taken during (t_k, t_{k+1}), if it is the case, $lbw_{q,a,k} = lw_{q,a,k-1}$; otherwise $lbw_{q,a,k} = 0$. Equations (11), (12),(13) and (14) summarizes $lw_{q,a,k}$ and $lbw_{q,a,k}$ calculations and (Figure 2) presents an example of these calculations.

$$lw_{q,a,0} = bw_{q,a,0} \;\forall\, q \in WS, a \in CU_q \qquad (11)$$

$\forall\, q \in WS, a \in CU_q, k \in \{2, \dots, KM - 1\}$:

$$lw_{q,a,k} = bw_{q,a,k} + lbw_{q,a,k} \qquad (12)$$

$$yw_{q,a,k} = 1 \rightarrow lbw_{q,a,k} = lw_{q,a,k-1} \qquad (13)$$

$$yw_{q,a,k} = 0 \rightarrow lbw_{q,a,k} = 0 \qquad (14)$$

	l_1	l_2	l_3	l_4	l_5
$yw_{q,1,k}$	1	1	1	1	0
$yw_{q,3,k}$	0	0	0	0	1
$bw_{q,1,k}$	l_1	l_2	l_3	l_4	0
$bw_{q,3,k}$	0	0	0	0	l_5
$lbw_{q,1,k}$	–	$yw_{q,1,2} \times lw_{q,1,1} = l_1$	$l_1 + l_2$	$l_1 + l_2 + l_3$	$yw_{q,1,5} \times lw_{q,1,4} = 0$
$lbw_{q,3,k}$	–	$yw_{q,3,2} \times lw_{q,3,1} = 0$	0	0	$yw_{q,3,5} \times lw_{q,3,4} = 0$
$lw_{q,1,k}$	$bw_{q,1,0} = l_1$	$bw_{q,1,2} + lbw_{q,1,2} = l_1 + l_2$	$l_1 + l_2 + l_3$	$l_1 + l_2 + l_3 + l_4$	$bw_{q,1,5} + lbw_{q,1,5} = 0$
$lw_{q,3,k}$	$bw_{q,3,0} = 0$	$bw_{q,3,2} + lbw_{q,3,2} = 0$	0	0	$bw_{q,3,5} + lbw_{q,3,5} = l_5$

Figure 2. Example of $lw_{q,a,k}$ and $lbw_{q,a,k}$ calculations

To be able to track the end of a product batch or a shutdown, the binary variable $ew_{q,a,k}$ indicates the end of processing the scheduling decision $a \in CU_q$in q during (t_k, t_{k+1}). $ew_{q,a,k} = 1$ if only if $yw_{q,a,k} = 1$ and $\sum_{z \in CU_a, z \neq a} yw_{q,z,k+1} = 1$, meaning that a is scheduled during (t_k, t_{k+1}) and another decision $z \neq a$ is scheduled during (t_{k+1}, t_{k+2}).

$\forall\, q \in WS, a \in CU_q, k \in \{1, \dots, KM - 2\}$:

$$ew_{q,a,k} \geq yw_{q,a,k} + \sum_{z \in CU_a, z \neq a} yw_{q,z,k+1} - 1 \qquad (15)$$

$$ew_{q,a,k} \leq \sum_{z \in CU_a, z \neq a} yw_{q,z,k+1} \qquad (16)$$

$$ew_{q,a,k} \leq yw_{q,a,k} \qquad (17)$$

For every product batch in a washing station or a secondary pipeline, a minimal batch size constraint should be respected as presented in equation (18). Furthermore, a minimal shutdown duration should be respected as a setup duration for the washing stations and as minimal stoppage duration for the secondary pipeline as in equation (19).

$\forall\, q \in WS, a \in CU_q \backslash \{PM + 1\}, k \in \{2, \dots, KM - 2\}$:

$$lw_{q,a,k} \times QR_{q,a} \geq ew_{q,p,k} \times VBWS_{q,p} \qquad (18)$$

$\forall\, q \in WS, k \in \{2, \dots, KM - 2\}$:

$$lw_{q,PM+1,k} \geq ew_{q,PM+1,k} \times WST_q \qquad (19)$$

4.4 *Volumes and products of batches in the slurry pipelines*

The sequence of the batches charged in slurry pipeline before the start of the scheduling horizon is known in advance and used to initialize the values of $y_{i,p}$. $p = PM + 1$ refers to a batch of water.

$$y_{i,p} = YO_{i,p} \;\forall\, i \in IO, p \in P + \{PM + 1\} \qquad (20)$$

Any new batch injected into the slurry pipeline during the scheduling horizon consists of at most one product or water. If i does not contain a product or water, it is considered as a fictitious batch.

$$\sum_{p=1}^{PM+1} y_{i,p} \leq 1 \;\forall\, i \in IN \qquad (21)$$

All fictitious batches are arranged after nonempty batches.

$$\sum_{p=1}^{PM+1} y_{i,p} \geq \sum_{p=1}^{PM+1} y_{i+1,p} \;\forall\, i \in \{IOM + 1, \dots, IM - 1\} \qquad (22)$$

No different products are allowed to be charged successively in the slurry pipeline. A batch of water must separate between different products as to avoid contaminations and to be able to detect batches extremities at the terminal station. Equation (23) satisfies this constraint.

$\forall\, i \in IN, p \in P + \{PM + 1\}, p' \in P + \{PM + 1\}$:

$$y_{i,p} + y_{i+1,p'} \leq 1 + FB_{p,p'} \qquad (23)$$

At the start of the scheduling horizon, the head station can continue to inject an additional quantity to the old batch IOM. IOM is the initialization batch that is closest to the origin of the slurry pipeline. The minimal and maximal batch size constraints for the batch IOM are presented in equations (24) and (25).

$\forall\, p \in P + \{PM + 1\}$:

$$\sum_{k=1}^{KM-1} vp_{1,IOM,p,k} + y_{IOM,p} \times WO_{IOM} \geq y_{IOM,p} \times VBPI_P \qquad (24)$$

$$\sum_{k=1}^{KM-1} vp_{1,IOM,p,k} + y_{IOM,p} \times WO_{IOM} \leq y_{IOM,p} \times VBPA_P \qquad (25)$$

The minimal and maximal batch size constraints for the rest of the batches are presented in equation (26).

$\forall\, i \in IN, p \in P + \{PM + 1\}$:

$$y_{i,p} \times VBPI_P \leq \sum_{k=1}^{KM-1} vp_{1,i,p,k} \leq y_{n,i,p} \times VBPA_{n,P} \qquad (26)$$

4.5 *Batch tracking in the slurry pipeline*

The volume of batch i in the slurry pipeline at moment t_{k+1} $(k < KM)$ is equal to the summation of the volume of the batch i at t_k ($w_{i,k}$), the volume injected to the batch i during (t_k, t_{k+1}) $(v_{1,i,k})$ and subtracting the volume discharged in the terminal station from batch i during (t_k, t_{k+1}). For t_{k+1}, where $k \in \{KM, \dots, KM + INM\}$, the charged volume is not taken into consideration as we are interested in fully discharging the batches present in the slurry pipeline without taking any charging decisions.

$\forall\ i \in I,\ k \in \{1, \dots, KM-1\}$:

$w_{i,k+1} = w_{i,k} + v_{1,i,k} - v_{2,i,k}$ (27)

$\forall\ i \in I,\ k \in \{KM, \dots, KM+INM-1\}$:

$w_{i,k+1} = w_{i,k} - v_{2,i,k}$ (28)

$w_{n,i,1}$ is initialized with the volumes of the initialization batches.

$w_{i,1} = wO_i\ \forall\ i \in IO$ (29)

The volumes of all the new batches at the start of the scheduling horizon in the slurry pipeline are initialized to zero.

$w_{i,1} = 0\ \forall\ i \in IN$ (30)

The upper coordinate of batch i in in the slurry pipeline is equal to the sum of the upper coordinate of batch $i+1$ and the volume of batch i.

$\forall\ i \in \{1, \dots IM-1\}, k \in \{1, \dots, KM+INM\}$:

$h_{i,k} = h_{i+1,k} + w_{i,k}$ (31)

The lower coordinate of the last batch IM charged into the slurry pipeline is equal to zero. Therefore, its upper coordinate must be equal to its volume.

$h_{IM,k} = w_{IM,k}\ \forall\ k \in \{1, \dots, KM\}$ (32)

The first batch $i = 1$ charged into the slurry pipeline is the farthest from the origin of the pipeline and its upper coordinate is equal the volume of the slurry pipeline.

$h_{1,k} = F\ \forall\ k \in \{1, \dots, KM+INM\}$ (33)

The upper coordinate of all batches in the slurry pipeline must be lower than its volume.

$h_{i,k} \le F\ \forall\ i \in I, k \in \{1, \dots, KM+INM\}$ (34)

4.6 Charging and discharging operations in the slurry pipeline

The duration of charging or discharging batch i in the slurry pipeline in the station j during (t_k, t_{k+1}) is calculated in equations (35) and (36) and its relative quantity is calculated in equation (39). Equations (37), (38) and (40) are used to calculate the quantity and duration of the discharging operations after the end of the charging scheduling horizon.

$\forall\ j \in J, i \in I, k \in \{1, \dots, KM-1\}$:

$x_{j,i,k} = 1 \rightarrow lx_{j,i,k} = l_k$ (35)

$x_{j,i,k} = 0 \rightarrow lx_{j,i,k} = 0$ (36)

$\forall\ i \in I, k \in \{KM, \dots, KM+INM-1\}$:

$x_{2,i,k} = 1 \rightarrow lx_{2,i,k} = l_k$ (37)

$x_{2,i,k} = 0 \rightarrow lx_{2,i,k} = 0$ (38)

$v_{j,i,k} = QIA \times lx_{j,i,k}\ \forall\ j \in J, i \in I, k \in \{1, \dots, KM-1\}$ (39)

$\forall\ i \in I, k \in \{KM, \dots, KM+INM-1\}$:

$v_{2,i,k} = QIA \times lx_{2,i,k}$ (40)

Only one product or water can be charged or discharged during a time interval (t_k, t_{k+1}) as in equation (9). Therefore, if $y_{i,p} = 1$, $vp_{j,i,p,k} = v_{j,i,k}$, otherwise $vp_{j,i,p,k} = 0$ as presented in the equations (41) to (44).

$\forall\ j \in J, i \in I, k \in \{1, \dots, KM-1\}$:

$v_{j,i,k} = \sum_{p=1}^{PM+1} vp_{j,i,p,k}$ (41)

$\forall\ j \in J, i \in I, k \in \{KM, \dots, KM+INM-1\}$:

$v_{2,i,k} = \sum_{p=1}^{PM+1} vp_{2,i,p,k}$ (42)

$\forall\ j \in J, i \in I, p \in P + \{PM+1\}$:

$\sum_{k=1}^{KM-1} vp_{j,i,p,k} \le y_{i,p} \times QIA \times TL$ (43)

$\forall\ i \in I, p \in P + \{PM+1\}$:

$\sum_{k=KM}^{KM+INM-1} vp_{2,i,p,k} \le y_{i,p} \times QIA \times TL$ (44)

The relation between the quantity that can be charged or discharged in the slurry pipeline and the length l_k of the time interval (t_k, t_{k+1}) is established in equations (45) and (46). If $\sum_{i=1}^{IM_n} x_{j,i,k} = 0$ (no batch is charged or discharged in the slurry pipeline), constraints (41) and (42) are redundant.

$\forall\ j \in J, k \in \{1, \dots, KM-1\}$:

$\frac{\sum_{i=1}^{IM} v_{j,i,k}}{QIA} \le l_k \le \frac{\sum_{i=1}^{IM} v_{j,i,k}}{QIA} + TL \times (1 - \sum_{i=1}^{IM} x_{j,i,k})$ (45)

$\forall\ j \in J, k \in \{KM, \dots, KM+INM-1\}$:

$\frac{\sum_{i=1}^{IM} v_{2,i,k}}{QIA} \le l_k \le \frac{\sum_{i=1}^{IM} v_{2,i,k}}{QIA} + TL \times (1 - \sum_{i=1}^{IM} x_{2,i,k})$ (46)

If the head station ($j = 1$) charges a batch i into the slurry pipeline during the time interval (t_k, t_{k+1}), batches $i' \in \{i+1, \dots, IM\}$ should not have been charged in the pipeline before t_k.

$\forall\ i \in \{\mathrm{IOM}, \dots, IM-1\}, k \in \{2, \dots, KM-1\}$: (47)

$x_{1,i,k} + \sum_{i'=i+1}^{IM} \sum_{k'=1}^{k-1} x_{1,i',k'} \le 1 + IM \times KM \times (1 - x_{1,i,k})$

If the head station injects batch i into the slurry pipeline during (t_k, t_{k+1}), the lower coordinate of batch i must be equal to be zero at t_k and t_{k+1} as presented in equations (48) and (49).

$\forall\ i \in IN, k \in \{1, \dots, KM-1\}$:

$h_{i,k} - w_{i,k} \le F \times (1 - x_{1,i,k})$ (48)

$h_{i,k+1} - w_{i,k+1} \le F \times (1 - x_{1,i,k})$ (49)

The head station cannot charge more quantities to the initialization batches except for the batch IOM that is closest the the origin of the slurry pipeline.

$\sum_{k=1}^{KM-1} x_{1,i,k} = 0\ \forall\ i \in \{1, \dots, IOM-1\}$ (50)

For every time interval (t_k, t_{k+1}), if the slurry pipeline is in a shutdown, no product is charged or discharged. Otherwise, one product at most can be charged in the head station and discharged in the terminal station.

$\sum_{i=1}^{IM} x_{1,i,k} = sp_k\ \forall\ k \in \{1, \dots, KM-1\}$ (51)

$\sum_{i=1}^{IM} x_{2,i,k} = sp_k\ \forall\ k \in \{1, \dots, KM+INM-1\}$ (52)

The stoppages of the slurry pipeline are rare and costly; therefore, the industrials prefer to schedule water if no product is charged in the slurry pipeline. Constraint (49) can be relaxed to take the pipeline stoppages into consideration.

$sp_k \ge 1\ \forall\ k \in \{1, \dots, KM+INM-1\}$ (53)

If a batch i is discharged in the terminal station during (t_k, t_{k+1}), its upper volumetric coordinate must be equal to the volume of the slurry pipeline at t_{k+1}.

$h_{i,k+1} \ge F \times x_{2,i,k}\ \forall\ k \in \{1, \dots, KM+INM-1\}$ (54)

4.7 Demand constraints at the terminal station

To satisfy the continuous demand for products $dc \in DC$, exactly the quantity defined by dc consumption flow rate needs to be delivered to its local consumer market.

$\forall\ dc \in dC, k \in \{1, \dots, KM+INM-1\}$:

$vmcp_{dc,k} = VMPD_{dc} \times l_k$ (55)

For products $dp \in DP$, at least the total demand must be satisfied before the end of the scheduling horizon.

$\sum_{k=1}^{KM+INM-1} vmcp_{dp,k} \ge VMPD_{dp}\ \forall\ dp \in DP$ (56)

At any time node, for products $dp \in DP$, the total quantity delivered to the filtration unit must not exceed its maximal processing capacity.

$\forall\ dp \in dP, k \in \{1, \dots, KM+INM-1\}$:

$vmcp_{dp,k} \le FFU \times l_k$ (57)

4.8 Inventory management constraints

The initial inventory level and the initial product stored in every tank s in every station m is prior given.

$vmt_{m,s,1} = VMTO_{m,s}\ \forall\ m \in M, s \in S_m$ (58)

$qst_{m,s,b,1} = OS_{m,s,b}\ \forall\ m \in M, s \in S_m, b \in CS_{m,s}$ (59)

Only and only one product can be stored in a tank s at t_k in the station m.

$\forall\ m \in \{1,2\}, s \in S_m, k \in \{1, \dots, KM\}$:

$\sum_{b \in CS_{m,s}} qst_{m,s,b,k} = 1$ (60)

$\forall\, m = 3, s \in S_m, k \in \{1, \dots, KM + INM\}$:
$$\sum_{b \in CS_{m,s}} qst_{m,s,b,k} = 1 \tag{61}$$
The change of stored product at t_k in a tank s can take place only if s is empty. The binary variable $avp_{m,s,k}$ can take the value 1 if the tank level $vmt_{m,s,k}$ is equal to zero in the station m at t_k. $avp_{m,s,k}$ is defined in equations (62) and (63).
$\forall\, m \in \{1,2\}, s \in S_m, k \in \{1, \dots, KM\}$:
$$avp_{m,s,k} \le 1 - \frac{vmt_{m,s,k}}{M} \tag{62}$$
$\forall\, m = 3, s \in S_m, k \in \{1, \dots, KM + INM\}$:
$$avp_{m,s,k} \le 1 - \frac{vmt_{m,s,k}}{M} \tag{63}$$
If a product b is stored in the tank s in the station m at t_k, a different product can be stored in s if only if $avp_{m,s,k+1} = 1$.
$\forall\, m \in \{1,2\}, s \in S_m, b \in CS_{m,s}, k \in \{1, \dots, KM - 1\}$:
$$qst_{m,s,b,k+1} \ge qst_{m,s,b,k} - avp_{m,s,k+1} \tag{64}$$
$\forall\, m = 3, s \in S_m, b \in CS_{m,s}, k \in \{1, \dots, KM + INM - 1\}$:
$$qst_{m,s,b,k+1} \ge qst_{m,s,b,k} - avp_{m,s,k+1} \tag{65}$$
A product b can be charged in the slurry pipeline only if it is stored in the multiproduct tank $s = 2$ at the head station.
$\forall\, b \in CS_{2,2}, k \in \{1, \dots, KM - 1\}$:
$$M \times qst_{2,2,b,k} \ge \sum_{i \in I} vp_{1,i,b,k} \tag{66}$$
If a new product b is discharged in the terminal station at t_k, the quality variable $qst_{3,2,b,k}$ is updated.
$\forall\, b \in CS_{3,2}, k \in \{1, \dots, KM - 1\}$:
$$M \times qst_{3,2,b,k} \ge \sum_{i \in I} vp_{2,i,b,k} \tag{67}$$
If a washing station or a secondary pipeline q discharges a new product in the tank $s = 2$ in the head station, the tank needs to change its quality.
$\forall\, b \in CS_{2,2}, k \in \{1, \dots, KM - 1\}$:
$$M \times qst_{2,2,b,k} \ge \sum_{q \in WS} \sum_{p \in CU_q \cap CS_{2,2}} AS_{q,2,2} \times yw_{q,p,k} \tag{68}$$
A product can be consumed from the tank $s = 2$ in the terminal station only if it is available.
$\forall\, b \in CS_{3,2}, k \in \{1, \dots, KM\}$:
$$vmcp_{b,k} \le M \times qst_{3,2,b,k} \tag{69}$$
The inventory levels of all the tanks are calculated in the following equations.
$\forall\, k \in \{1, \dots, KM - 1\}$:
$$vmt_{1,1,k+1} = vmt_{1,1,k} + bw_{1,1,k} \times QR_{1,1} - bw_{4,1,k} \times QR_{4,1} \tag{70}$$
$$vmt_{2,1,k+1} = vmt_{2,1,k} + \sum_{q=2}^{4} bw_{q,1,k} \times QR_{q,1} - \sum_{i \in I} vp_{1,i,1,k} \tag{71}$$
$$vmt_{2,2,k+1} = vmt_{2,2,k} + \sum_{p=2}^{PM} bw_{3,p,k} \times QR_{3,p} - \sum_{p=2}^{PM} \sum_{i \in I} vp_{1,i,p,k} \tag{72}$$
$\forall\, k \in \{1, \dots, KM + INM - 1\}$:
$$vmt_{3,1,k+1} = vmt_{3,1,k} + \sum_{i \in I} vp_{2,i,1,k} - vmcp_{1,k} \tag{73}$$
$$vmt_{3,2,k+1} = vmt_{3,2,k} + \sum_{p=2}^{PM} \sum_{i \in I} vp_{2,i,p,k} - \sum_{p=2}^{PM} vmcp_{p,k} \tag{74}$$
The inventory of every product in every station must be always kept within the permissible range.
$\forall\, m \in \{1,2\}, s \in S_m, k \in \{1, \dots, KM\}$:
$$VMTI_{m,s} \le vmt_{m,s,k} \le VMTA_{m,s} \tag{75}$$
$m = 3, s \in S_m, k \in \{1, \dots, KM + INM\}$:
$$VMTI_{m,s} \le vmt_{m,s,k} \le VMTA_{m,s} \tag{76}$$

5 CASE STUDY

The research presented in this paper is motivated by a real-world case study of a pipeline network and washing stations scheduling problem of the OCP Group. The OCP Group is a Moroccan state-owned company that specializes in mining, producing, and manufacturing phosphate-based products. In particular, the OCP has built a slurry pipeline with a length of 187km, a total volume of 106000 m^3 and a flow rate of 4000 m^3/h to transport large volumes of slurry from the head station to the terminal station.

In order to evaluate the performance of the proposed model, a group of 10 scenarios were generated using real dataset from the OCP Group's supply chain. These scenarios involve a scheduling horizon of the slurry pipeline's charging operations of $TL = 36\,h$, and a total scheduling horizon of $62.5\,h$ considering all of discharging operations at the terminal station. Instances are created using the following criteria: The number of products $PM \in \{1, 2, 3\}$, the initial inventory and the capacity of storage tanks and the clients' demands during the planning horizon. Please note that the water is used as separator product that is not included in PM. All instances and results presented in this section are available at: https://zenodo.org/record/7850304#.

The number of time nodes KM is prefixed for each scenario according to historical data. The maximum value of KM is chosen based on an empirical experiment and it depends on the number of products and the length of time horizon. The number of batches IM=$IOM + INM$, where IOM is the number of old batches (initial state of the slurry pipeline) and INM is the maximum number of new batches. The numerical experiments were conducted on a 64 threads CPU (AMD EPYC 7452 32-Core Processor) with 500 GB of RAM under Linux (Ubuntu 20.04.5), and the instances were solved using Gurobi 9.5.

Table 1. Obtained results

Inst.	Nb. of products (PM)	Nb. of time nodes (KM)	Nb. of batches ($IOM + INM$)	MILP model	
				Obj. Function	CPU time (s)
1	3	17	6+9	95.83%	68.49
2	3	17	7+9	91.15%	274.89
3	3	17	3+9	95.83%	326.36
4	3	17	6+9	92.14%	279.46
5	2	15	3+8	95.83%	7.74
6	2	15	5+8	94.82%	36.32
7	2	15	5+8	93.23%	62.62
8	1	10	3+5	94.44%	0.47
9	1	10	3+5	94.44%	1.81
10	1	10	3+5	94.44%	1.91

As shown in Table 1, the objective value varies from 91.15% to 95.83% for the group of instances. The difference between objective values is mainly from the number of scheduled batches (new batches). For instances 8, 9 and 10, the same objective value can be explained by the same number of batches with identical volume of water in obtained solutions. In term of CPU Time, the proposed model provides optimal solutions for all instances within less than 6 minutes. Furthermore, Figure 3 and 5 illustrate the storage levels for products in the terminal station for the solution of instance 1. Figure 4 shows the scheduling of operations of the slurry pipeline, the secondary pipeline 1 and the washing stations with all of related time nodes.

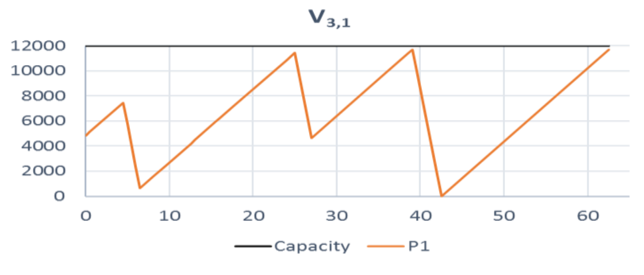

Figure 3. Inventory level of $V_{3,1}$ of instance 1

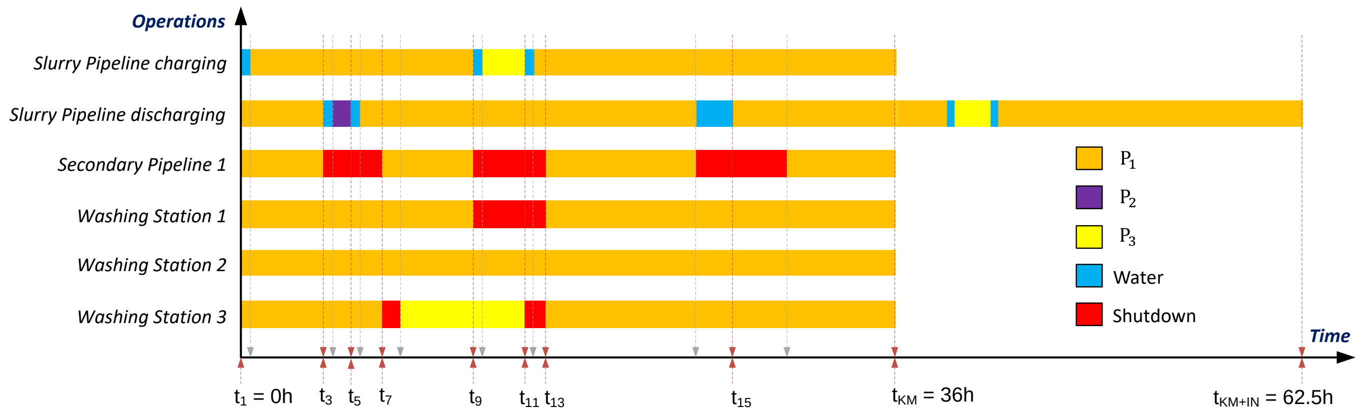

Figure 4. Scheduling solution of instance 1

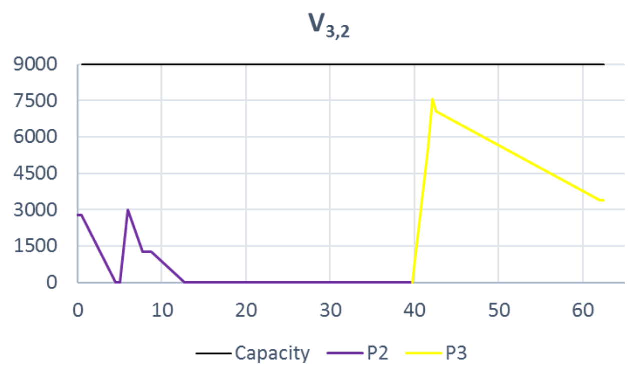

Figure 5. Inventory level of $V_{3,2}$ of instance 1

6 Conclusion

This paper presents a MILP base approach to optimize the detailed scheduling solution for the integrated production (washing stations) and multiproduct pipelines transportation scheduling problem in the context of phosphate mining industry. In addition, a detailed storage management for multiproduct tanks was also considered. The proposed MILP model aims to maximize the overall utilization rate of the main pipeline for phosphate slurry transportation, by considering the constraints related to production, transport, storage capacity, and satisfaction of customers' demand requests. The proposed model was evaluated using a group of 10 instances varying in complexity. The obtained results show that the model is able to provide the optimal solution for all instances in a reasonable amount of time.

Future work will concentrate on considering all the other operational constraints in the pipeline network of the OCP's supply chain, such as the introduction of a new transfer secondary pipeline between storage tanks at the terminal station. Later, a heuristic approach should be developed for solving instances with longer planning horizon in future research.

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