Modelling the Mathematical Behaviour of a Computerized Hierarchical Questionnaire

Martin Paradis & Sylvain Delisle
Institut de recherche sur les PME
Département de mathématiques et d’informatique
Université du Québec à Trois-Rivières
Trois-Rivières, Québec, Canada, G9A 5H7
Sylvain_Delisle@uqtr.ca    www.uqtr.ca/~delisle

Abstract
We present a prototype model enabling us to study the mathematical behaviour of a computerized hierarchical questionnaire. Besides its hierarchical structure, the latter has the following characteristics: i) a closed set of pre-determined acceptable answers is associated to every question, and ii) the questionnaire should return a certain numerical outcome based on the answers, each of these being associated with a numerical weight. The model uses a program called a results generator, which generates all possible results; thus the requirement to have a finite number of possible answers. The generator transforms the questionnaire into a mega-decision tree to process every possible result from which we derive a curve. From that curve (a depiction of a mathematical function), we propose a way to interpret the results. The model is applied to a real-life computerized hierarchical questionnaire to demonstrate its capabilities.

Keywords: Simulation, modeling methodologies, (computerized) hierarchical questionnaire, calculation model, information visualisation.

1. Introduction

In this paper, we discuss a prototype model we have developed to study the mathematical behaviour of a computerized hierarchical questionnaire. The need to develop such a model arose from the creation of a software aiming to help a user to analyse the risk level of a SME (Small and Medium-sized Enterprise) project. This software, eRisC [2, 3], was developed for and validated by entrepreneurs, economic agents, lenders and investors, to identify the main risk factors of SMEs development projects in order to improve their success rate and facilitate their financing.

The model we here propose is an extension to the eRisC software. In order to evaluate the risk level associated with an SME project, eRisC uses three relatively complex calculation models. However, as these calculation models emerged from the development of the hierarchical questionnaires used by the software (see Section 2), essentially as a by-product rather than being explicitly devised at the outset, the numerical results returned by the tool (i.e. the risk level) turned out to be difficult to interpret from the user’s standpoint. In order to have a better understanding of the results returned by the eRisC software and the implied computation models used in its inner core, we had to develop a methodology to do so and this is what we present here. Despite an extensive search of the literature, we were surprised to discover that such a problem and its potential solutions had not been documented yet. Because of this fact, and also because of the specific application domain we are dealing with, we claim that our work appears to be quite original.

This paper is divided in three sections. Section 2 provides more detail on the eRisC tool. Section 3 gives a complete description of the model we have developed. This model is mostly based on a program we have created and which is called the (results) generator. Section 4 describes the results obtained by
the generator for a real SME project, before we conclude in Section 5.

2. The eRisC System: Risk Assessment of SME Development Projects

2.1 A Quick Overview of the eRisC System

The eRisC system identifies, measures and allows to manage the main risk factors that could compromise the success of SMEs development projects including expansion, export and innovation projects, each of which is the object of a separate questionnaire in the software. An extensive dynamic, Web-based questionnaire is used to collect relevant information items on the SME project to be evaluated. This tool benefited from the most recent Web-based technologies (e.g. Oracle Java) and was right from the start designed as a fully automated system. Actual screens appear on the next page (Figure 2a,2b) to illustrate eRisC’s user interface.

2.2 What is a Hierarchical Questionnaire?

All three questionnaires (i.e. expansion, export, innovation) used by the eRisC software are hierarchical questionnaires. First of all, embedded levels of questions characterize this type of questionnaire. Depending on the answer selected by the user (from a closed list of a maximum of four potential answers in our case, see Figure 2a), a question may generate sub-questions. Thus, a level 1 question may or may not generate sub-questions of level 2, depending on the answer, and so forth. eRisC contains up to 7 levels of such sub-questions. Secondly, all questions of a hierarchical questionnaire possess a predefined, finite set of acceptable answers. This is important because the results generator we present in Section 3 requires that we generate (i.e. compute) all potential results. To do so, we need to have a finite, although eventually quite large, number of possible results. Finally, to every answer must be assigned a numerical value or weight. In the eRisC software, a weight system is used to calculate the global risk level of the evaluated project: this risk level, a numerical value in the [0,100] range, constitutes the final output of the software—Figure 2b shows an example result of 42.22.

2.3 Calculation Models for Questionnaires

Each of the three hierarchical questionnaires (expansion, export, and innovation) possesses a different underlying calculation model. These elaborated knowledge-intensive algorithms are closely related to the questionnaires’ embedded structure. The details of these models go beyond the scope of this paper: what matters here is that each model computes the risk level of a project according to 1) the specific questionnaire used, and 2) the evaluator’s (user’s) answers.

eRisC’s computation model also identifies the “heaviest” risk elements associated with the evaluated project and is able to rank them in reverse order of importance so that the user can eventually consider ways to mitigate them in order to reduce the project’s level of risk.

3. The Results Generator

To better understand the behaviour of the different questionnaires and their implied computation models, we first needed to study the global behaviour of the results themselves. In order to do so, we created a program called the results generator that enables us not only to determine the actual range of all the possible results returned by eRisC, but also to find out the frequency of every result. These results allow us to derive a curve (one per questionnaire) of their frequency. In other words, these curves give us the probability of obtaining a given result, for a specific questionnaire. It is possible to build such a generator program based on the fact that there are a finite number of potential results; this is because the user has a predefined (closed) set of possible answers for each question. At the beginning, we had absolutely not idea about how the resulting curves would look like (see Figure 1 below): only that they would help us to better understand the implicit calculation model within each questionnaire.

---

1 https://oraprnt.uqtr.uquebec.ca/erisc/index.jsp

Figure 1. Examples of hypothetical curves for one of eRisC’s questionnaires, before the actual computations with the results generator. We call these GCR (global curve of results) curves.
Figure 2a. An excerpt from eRisC’s expansion project questionnaire. The only acceptable answers to questions are YES, NO, NOT APPLICABLE, DON’T KNOW.

Figure 2b. Risk Assessment Results Produced by eRisC. Here the global risk level is 42.22.
We were hoping to be able to draw meaningful and useful conclusions according to the resulting curves obtained with the help of the generator. In particular, we were hoping that the curves would help us visualise the distribution (or density) of the potential risk-level results returned by eRisC which were supposed to be in the [0,100] range. For example, is the user more likely to obtain a low risk-level result, a medium risk-level result or a high risk-level result, or does the risk evaluation model give an equal chance to obtain any risk-level result? As of now, the risk level is presented to the user as a quantitative result as shown in Figure 2b above (i.e. 42.22). This value has no significance whatsoever to the user other than “the higher it is, the higher the risk level is”. Thus, instead of saying that the total project risk-level is, say, 42.22, we may prefer to say that the total project risk-level is “medium-high”, for instance, assuming that the latter is more intuitive and meaningful to the user.

Not only did we not clearly understand the behaviour of eRisC’s underlying calculation models at the beginning of our work, but we also identified certain factors that could affect the risk level of an evaluated project. We consider these factors as “uncertainty factors” [1, 5, 6]. Here, the uncertainty consists of factors that may influence in some way the end result (i.e. risk level) such as the expertise of the evaluator, her/his global understanding and appreciation of eRisC, or her/his level of comprehension of the project itself, etc. However, details on these factors go beyond the scope of this paper and we will not elaborate any further, although they are important and are considered in [4]. Nevertheless, we need to find a way to study the impact of such factors on eRisC’s evaluations. To do so, we have decided to create a series of prototype projects that will enable us to analyse more precisely the effect of “uncertainty” on the risk level. We called upon risk-assessment experts to develop such prototype projects.

Different types of projects need to be created for this matter, and all projects should be evaluated by a number of evaluators (a minimum of 12 is recommended). It is also important to choose evaluators with different backgrounds, depending on the uncertainty factors we want to evaluate. For example, if we want to study the effects of the evaluator’s expertise, we should choose evaluators with different levels and areas of expertise. Although it is impossible to measure precisely the impact of each uncertainty factors on the risk level computed by eRisC, we should at least be able to estimate to what extent a given factor will affect the end result by comparing the results of each evaluator.

Other prototype projects could be used to study the different natures of evaluated SME projects. We know that even though projects are of the same type (expansion, export, innovation), the interpretation of their risk level can change depending on the specific nature of the project. For example, we hypothesize that a risk level of 67.41 could be considered “high” for expansion project A, while it could be considered “medium” for expansion project B. For instance, let us assume a risk-level distribution curve such as that of Figure 3 below. We can clearly see that, depending of the nature of the project, the risk-level of 67.41 may be considered “high” for a certain project and “medium” for another. This scaling is easier to interpret and also more meaningful than absolute numbers such as 67.41.

![Figure 3. Hypothetical curves representing the risk levels “low” (left), “medium” (center) and “high” (right) for projects A and B. For each, a scale shows the potentially different interpretations, “high” (above) or “medium” (below), for a given risk-level value (67.41) relative to the nature of the project.](image)
hand, perceptual questions call upon the expertise of each evaluator and their own personal assessment abilities. Therefore, the answers to these questions are subject to more variation than for factual questions.

As an experiment to evaluate our hypothesis and modelling approach, we then ran the results generator on a prototype project prepared by experts and based on a real-life SME project. Every time the generator encounters a factual question, it only takes into consideration the “correct answer” as provided by the experts. When a perceptual question is encountered, the generator considers all the potential answers for this question. In the end, we generated the curve representing the prototype project, which we then compared to the global curve of results (GCR) of eRisC (for the same questionnaire, of course).

4. Application of the Results Generator on a Real Project

We have tested the model proposed in the previous section on an SME project especially prepared for this purpose. Because of time constraints, only one project for eRisC’s expansion questionnaire was tested. The results generator works for the expansion and the export questionnaires, but not yet for the innovation questionnaire—the innovation questionnaire is based on an entirely different computation algorithm and will require that the results generator be adapted to proceed similarly.

The number of questions the user has to answer ranges from 59 to 93 for an expansion project, from 58 to 149 for an export project, and from 86 to 216 for an innovation project. And for each question, only four pre-determined answers are allowed (see Figure 2a above).

For the expansion questionnaire, we proved that there are 6.53e+44 different potential risk-level results, while there are 3.83e+43 for the export questionnaire. In other words: if we look at these two hierarchical questionnaires as decision trees, which indeed they are equivalent to, the expansion decision tree contains 6.53e+44 different branches, while the export decision tree contains 3.83e+43 different branches!

The first thing we noticed when we started to analyse the results we obtained with the generator is that the range of possible results does not vary between 0 and 100 as we first thought when we developed the questionnaires. For the expansion questionnaire, the results range between 0 and 89, and between 0 and 93 for the export questionnaire. Because of this, we now know that the risk-level values returned by eRisC are not percentages as we first hoped. A simple adjustment calculation would allow us to bring back the result in the [0,100] range if this is what we think is best. The resulting curve (see Figure 4 below) allows us to visualise the global behaviour of eRisC’s calculation model.

We can observe that for the expansion questionnaire, almost all of the risk-level values, 99.29% to be precise, are between 20 and 48. In other words, it is nearly impossible that the risk-level value be outside the range [20,48]. That range is [25,55] for the export questionnaire. This could mean, amongst other things, that the questions’ weights may need to be changed in order to widen the attainable range.

Furthermore, the resulting curves from both expansion and exportation questionnaires resembles that of the normal law, so well-known in statistics. We have performed detailed calculations and the fitting is almost perfect: so much that we decided to consider the curves as such. Using this normal law hypothesis, we used the standard deviation to separate each curve into three inner levels of risk: “low”, “medium”, and “high”. Thus, for the expansion questionnaire, the ranges are respectively [0,28.52] (“low risk”), [28.52,39.48] (“medium risk”), and [39.48,89] (“high risk”). Although we have chosen the standard deviation as an experimental reference value, the resulting ranges seem to be quite accurate and do fit the data we have obtained so far.

Once both GCR were generated, we then analysed the first prototype project with the expansion questionnaire. Fourteen human evaluators of different areas of expertise and different experience levels were asked to complete the expansion questionnaire for the very same project. Figure 5 below shows the resulting curve from the project. We remind the reader that contrary to the GCR, the project curve differentiates between factual and perceptual questions. We first note that, theoretically, the results for this project should be between 17 and 43. We say theoretically because the evaluators have not necessarily given the “correct answers” to all the factual questions, as errors are always possible. Also remember that the “correct answers” are the answers given to us by the experts
who prepared the prototype project for all the perceptual questions.

For the 14 evaluators, the average risk-level obtained for this project was 42.4. We then asked the evaluators to give us their “ball-park” qualitative estimation on the risk level of the project without providing any feedback on the results they had obtained. The risk level of this specific prototype project was considered to be between medium and medium-high, which corresponds to the intervals established earlier with our model and which provides support to our normal law hypothesis. With this project, we have also experimented with two uncertainty factors: the expertise and the experience of the evaluator. We found out that these factors did not influence very much the risk-level of the evaluated prototype project.

---

**Figure 4.** Comparison between eRisC’s expansion and export questionnaires.

**Figure 5.** Comparison between the GCR of the expansion questionnaire and the prototype project’s curve.
Obviously, before making any strong statement with regard to the validity of the model we have proposed here, further experiments are needed. However, it has proven very useful in our case and the results we have obtained so far do match the data. Plus, we have gained a level of understanding with eRisC’s “hidden” calculation model that we never had before. Also, the GCR for the expansion and exportation questionnaires both fit the normal law: this comes as no surprise since their calculation models are essentially the same. On the other hand, the innovation questionnaire has a different and more complex calculation model which may result in a very different GCR. Only future work will tell.

5. Conclusion

In this paper we proposed a model and methodology enabling us to study the mathematical behaviour of a computerized hierarchical questionnaire, a first step in the understanding of eRisC’s “hidden” risk evaluation model.

The study was conducted in two main steps. The first step consisted in computing the curve representing all possible results for a given questionnaire, thus obtaining their distribution. This allowed us to determine the range of possible results as well as the dispersion and frequencies of the results. The second step used prototype projects in order to study the impacts of projects’ specifics on the evaluation scheme, as well as the impact human evaluators (i.e. eRisC’s users) may have on risk assessment. This also allowed us to study certain “uncertainty” factors that may influence the final result computed by eRisC’s questionnaires.

In particular, for eRisC’s expansion and export questionnaires, we proved that they were equivalent to decision trees containing, respectively, 6.53e+44 and 3.83e+43 branches. We also proved that the risk-level results were in the [0,89] range for the expansion questionnaire and in the [0,93] range for the export questionnaire—both different from the expected [0,100] range. However, for each one, 99% of the potential results appear in a much narrower range: [20,48] and [25,55], respectively. We also proved that the distribution of their result values both fitted that of the normal law and this was very useful to determine to what level of risk (low, medium, high) a risk value may be said to correspond, for the user, in order to facilitate results interpretation with the eRisC software.

Up to now, two out of three questionnaires have been analyzed with the model proposed here. Thus, future work will consist in completing the experimentation with the innovation questionnaire, including the necessary adjustments to the results generator to carry out this task. We plan to continue the evaluation of projects to further our comprehension of the model in order to determine if the conclusions we have drawn so far continue indeed to match practical results obtained from actual SME project assessments. Thus far, results from this research project are quite encouraging.

References